183. **Proposed by M. T. GOODRICH, Dixfield, Maine.**

Show what relation must exist between the quantities A, B, and C, in the harmonic ratio $\frac{AB}{(A+B+C)(-C)}=-1$, so that they will be real positive integers.

Solution by S. LEFSCHETZ, Clark University.

This can be written $AB=(A+B+C)C$. Let δ be the greatest common divisor of A, B, C, and let $A=\delta A'$, $B=\delta B'$, and $C=\delta C'$. Substituting we have $A'B'=C'(A'+B'+C')$.

Let now $C'=p.q$, p being prime to B', and q to A'. Since p divides $A'B'$ and is prime to B', it divides A'. Let $A'=\lambda p$, and similarly $B'=\mu q$. Since A', B', pq have no common divisors, q must be prime to λ and p to μ. We have by substitution, $\lambda\mu=\lambda p+\mu q+pq$, or $2pq=(\lambda-q)(\mu-p)$, and p being prime to $\mu-p$, while it divides the right member, must necessarily divide $\lambda-q$. Let $\lambda-q=hp$, similarly, $\mu-p=kq$.

$\therefore hk=2$. Hence, either $h=1$ and $k=2$, or $k=1$ and $h=2$.

On account of the symmetry in notations it is sufficient to consider the case $h=1$, $k=2$. Then $\lambda=p+q$, $\mu=p+2q$.

$\therefore A=\delta p(p+q)$, $B=\delta q(p+2q)$, $C=\delta pq$, is the general solution, p and q being prime to each other. This can be verified easily by substitution.

Also solved by the Proposer.

AVERAGE AND PROBABILITY.

202. **Proposed by F. P. MATZ, Ph. D., Reading, Pa.**

If three chords are drawn at random in a circle, what is the chance the center of the circle is enclosed by the three chords, and what is the mean area of this enclosing triangle?

Solution by the late G. B. M. ZERR, Ph. D.

Let AB, CD, EF be the chords intersecting in Q, P, and R, respectively. Let O be the center. Draw the perpendiculars OG, OH, OL to AB, CD, EF, respectively. Also draw OA, OB, OC, OD, OE, OF, OQ, OP, OR.

Let $\angle AOG=\theta$, $\angle COH=\phi$, $\angle EOL=\psi$, $\angle HOG=\rho$, $\angle HOL=\mu$, $\angle LOG=\omega$, $\angle POQ=\delta$, $\angle QOR=\gamma$, $OP=x$, $OQ=y$, $OR=z$, $a=$radius of circle. The limits of $\theta=0$ and $\frac{1}{2}\pi$; of ϕ, 0 and θ; of ψ, 0 and ϕ; of ρ, $2\pi-2\psi-\theta-\phi=\rho'$ and $\pi-\theta-\phi=\rho''$; of μ, $2\pi-2\theta-\phi-\psi=\mu'$ and $\pi-\phi-\psi=\mu''$; of ω, $2\pi-2\phi-\theta-\psi=\omega'$ and $\pi-\theta-\psi=\omega''$; of δ, 0 and π; of γ, 0 and π; of x, 0 and a; of y, 0 and a; of z, 0 and a. Let $A=$average area, $C=$the required chance.

$$\therefore C=\frac{\int_0^{\frac{1}{2}\pi}\int_0^{\theta}\int_0^{\phi}\int_{\rho''}^{\rho'}\int_{\mu''}^{\mu'}\int_{\omega''}^{\omega'} d\theta\, d\phi\, d\psi\, d\rho\, d\mu\, d\omega}{\int_0^{\frac{1}{2}\pi}\int_0^{\theta}\int_0^{\phi}\int_0^{\pi}\int_0^{\pi}\int_0^{\pi} d\theta\, d\phi\, d\psi\, d\rho\, d\mu\, d\omega}$$